Supplemental material of "Distributed Shortcut Networks: Low-latency Low-degree Non-random Topologies Targeting the Diameter & Cable Length Trade-off"

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1 Further related work with small-world models

In Section 2 of our paper, we share some common techniques in design with some traditional regular topologies, such as the DLN-2^x topology in [1], [2], the CCC topology, or butterfly networks: the fashion of our selection of shortcuts comes from a super node that may resemble the 2^x links in the mentioned DLN or the dimension links between adjacent nodes in a hypercube. However the designing philosophy we use is different, because it is learned from observing small-world networks, i.e., Kleinberg's model [3].

As a subject of extensive studies, small-world networks have been introduced to model network structures of popular real-world large-scale networks [4]. Typically, a small-world model is a simple regular topology such as a torus with added random links. In Watts and Strogatz's model, a ring is augmented with random links: with a given probability parameter p, each local ring link is considered to be replaced by a uniform random link [4]. This work drew a lot of attention on small-world models, where just a few added random links can drastically reduce the diameter. Below we briefly review some existing topologies, which are inspired by small-world networks.

1) Random Shortcut Networks [1]: Typically, (uniform) random shortcuts are added to a ring of nodes. RSN(n,k) is formed based on a ring of n nodes, such that every pair of nodes have the same probability to have a shortcut connecting them, and the degree of all the vertices of the resulting graph is the same k.

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2) For DLN-2^x with a given degree x, we reuse a special Distributed Loop Network description from [2]: n vertices are arranged in a ring, and a shortcut is added between vertices i and j such that $j = i + \lfloor n/2^k \rfloor \mod n$ for $k = 1, \ldots, x-2$. For $x = \log n$, obviously, for any given destination node t, any other node u has at least one shortcut link that halves the distance to t. Thus, it is easy to see that this graph has a logarithmic diameter.

These long-range shortcuts have made possible the distance-halving technique, which is widely used in analyzing graphs as well as designing topologies with small diameter. Generally, in these graphs, when a node u searches for a path to another node v, it will find a long link from itself or a nearby node that goes closer to v by at least half of the (u, v) distance (for a given predefined distance metric). As mentioned, DLN-2^x with $x = \log n$ has a logarithmic diameter and also a natural simple routing logic, but has degree $\log n$, which is not as low as desired.

Now we look closer at Kleinberg's small-world network [3]. In a base $n \times n$ grid graph, each node has at most 4 links to the neighbor nodes. A random shortcut is attached to each node, where the node u's shortcut goes to another node v with the probability inversely proportional to the square of the lattice distance between u and v. Kleinberg's small-world network has short cable length, with constant degree and logarithmic diameter. This graph features the small-world effect in full: short routes between any two nodes are abundantly available, and moreover, any node u can find a path of length $O(\log^2 n)$ to any other node v by using local information only. This is done by greedy routing, which works naturally based on the distance-halving technique. However, greedy routing can only find paths of length $\theta(\log^2 n)$ [5], asymptotically quadratic of the minimum.

References

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